Probability distribution and first passage properties of evanescent run-and-tumble particle

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INTRODUCTION

Run-and-tumble particles (RTP) are self-propelling particles that take up energy from their environment and convert some of the energy into motion [1]. Its motion consists of alternating sequence of straight line motion nearly at constant speed which is called *run* followed by a sudden change in direction called *tumble* [2]. Bacteria such as *Escherichia coli*, *Salmonella typhimurium* and certain algae are examples of microorganisms doing run-and-tumble motion [3]. Currently, there have been a surge of interest in the first passage time problem and the related absorption problems involving run-and- tumble particles [1-4]. The first passage time provides the information of the average time needed for a particle to reach a particular site or target for the first time and has been used to analyze search processes, chemical reactions, intracellular transport and financial time series analysis [5]. With the presence of absorbing or reflecting boundaries, the relevant quantity now represents the time the particle reacts with the boundaries [6]. Most of the mentioned studies above have not considered the possibility that the run-and-tumble particles may die, decay or evanese which is a common occurrence in all living systems. For example, *E.coli* bacteria and other organism may die by programmed cell death [7]. Hence, the aim of this presentation is to provide the probability distribution and first passage characteristics of evanescent run-and tumble particles in one dimension. Evanescent run-and-tumble particles are run-and-tumble particles that may die, decay or evanesces while in motion.

EVANESCENT RUN-AND-TUMBLE PARTICLE

We consider an evanescent run-and-tumble particle that runs with velocity v, tumbles with rate α and die or evanesces with a rate λ. The system is confined to a partially reflecting (partially absorbing) region $a \leq x \leq b$ [6]. We represent the probability density function (PDF) for the right-oriented and left-oriented particles as $P_R(x, t)$ and $P_L(x, t)$ respectively

$$\frac{\partial P_R}{\partial t} = -v\frac{\partial P_R}{\partial x} + \frac{\alpha}{2}P_R - \frac{\alpha}{2}P_L - \lambda P_R \qquad (1)$$

$$\frac{\partial P_L}{\partial t} = v\frac{\partial P_L}{\partial x} - \frac{\alpha}{2}P_R + \frac{\alpha}{2}P_L - \lambda P_L \quad . \qquad (2)$$

From equations (1) and (2), the following steps are done, as shown in Figure 1, to obtain the probability distribution and conditional mean first passage time:

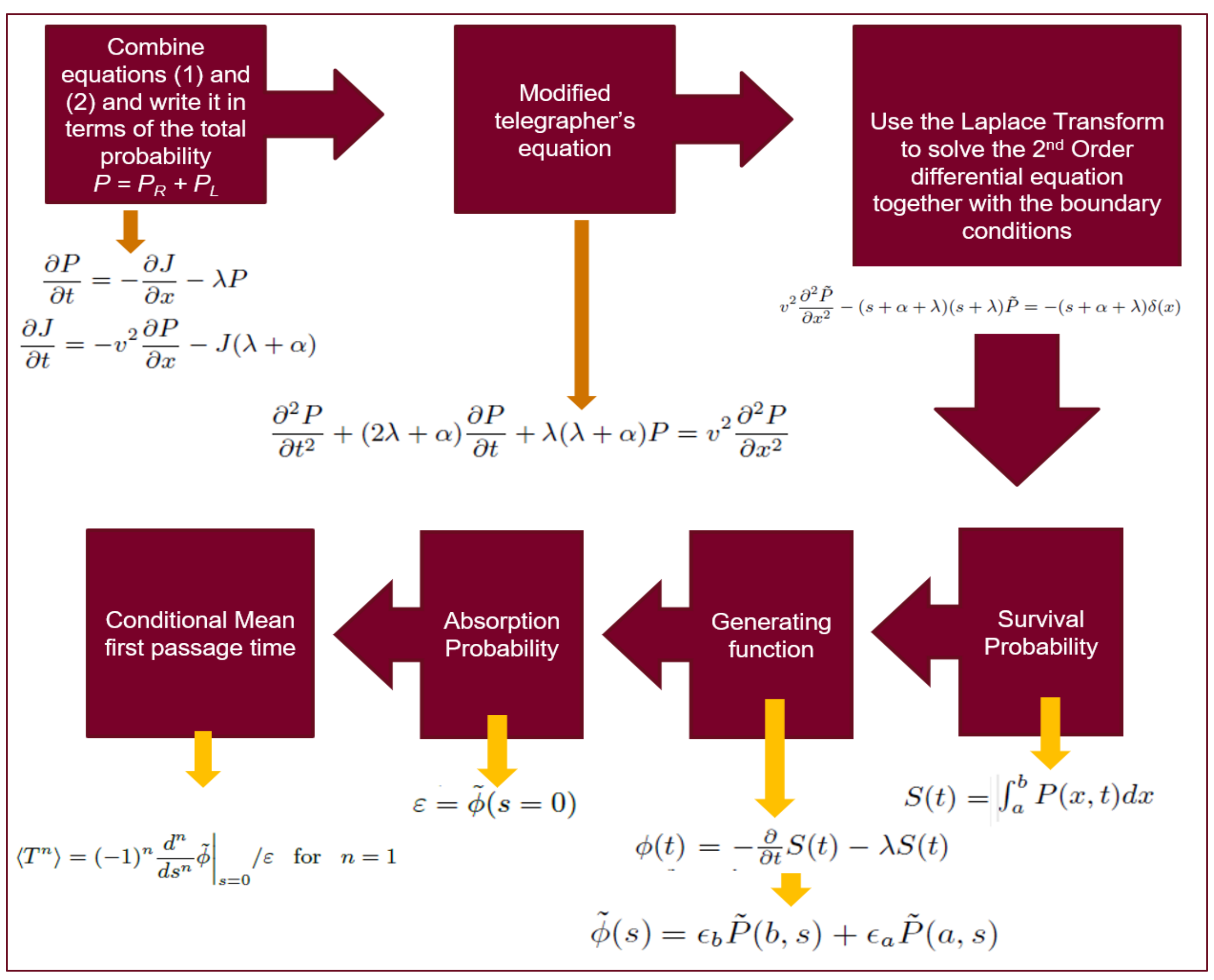

Figure 1: Schematic representation for obtaining the first passage properties

PROBABILITY DISTRIBUTION AND FIRST PASSAGE PROPERTIES

The probability distribution of an evanescent run-and-tumble particle in a symmetric boundaries with $a = -b = L$ is obtained in the Laplace space and is given by the equation below

$$\tilde{P}(x,s) = \frac{s+\alpha+\lambda}{2kv}\left\{\frac{k(e^{\frac{k}{v}(x-L)} + e^{-e^{\frac{k}{v}(x-L)}}) - \epsilon\,(s+\alpha+\lambda)(e^{e^{\frac{k}{v}(x-L)}} - e^{-e^{\frac{k}{v}(x-L)}})}{k(e^{\frac{k}{v}L} - e^{-\frac{k}{v}L}) + \epsilon\,(s+\alpha+\lambda)(e^{\frac{k}{v}L} + e^{-\frac{k}{v}L})}\right\}$$

where $v^2k^2 = (s+\alpha+\lambda)(s+\lambda)$. ϵ is the reflecting or absorbing coefficient. We can observed a normal distribution centered at $x = 0$ and an increase in the peak of the distribution with increasing death rate as shown in Figure 2.

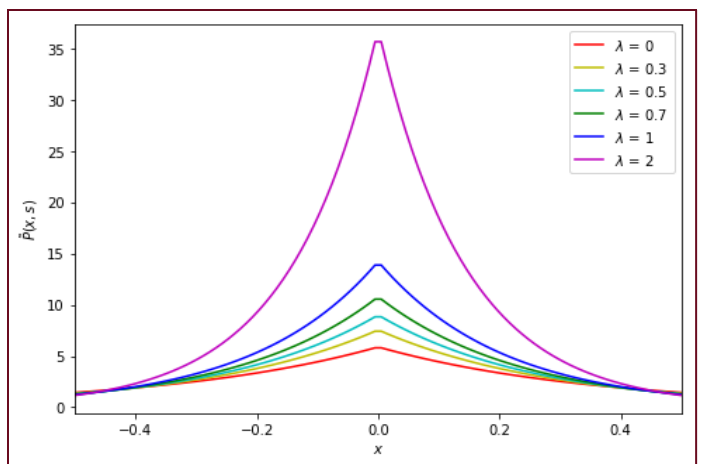

Figure 2: Probability distribution of evanescent RTP

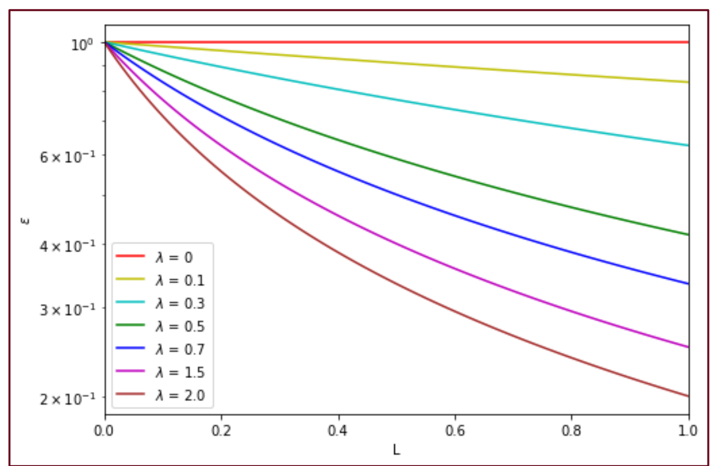

Figure 3: Eventual absorption probability of evanescent RTP [8]

The eventual absorption probability ε is obtained from the generating function by setting $s = 0$, which gives us

$$\varepsilon = \frac{2\epsilon}{2\epsilon + 2b\lambda/v + \epsilon b^2\lambda(\alpha+\lambda)/v^2}.$$

The behavior ε is shown in figure 3. While most the RTP model with λ=0 has an absorption probability of $\varepsilon = 1$, it can be observed that the eventual absorption probability of evanescent RTP decreases as the death rate increases. This indicates that not all of the particles are absorbed as some may have already died before reaching the boundary.

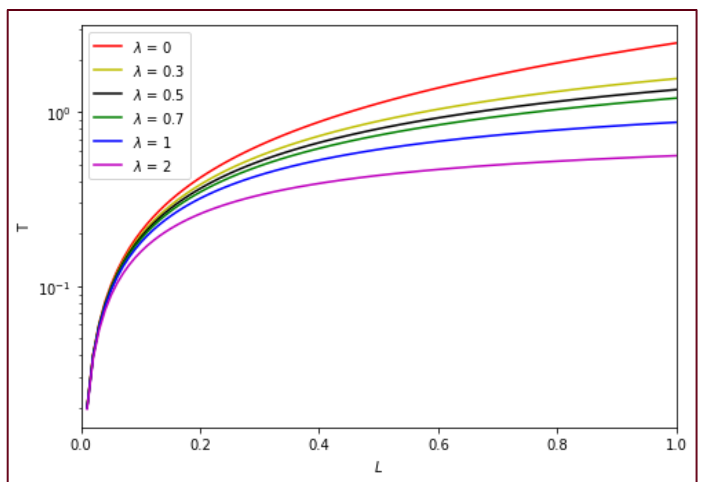

Figure 4: Conditional mean first passage time at constant velocity

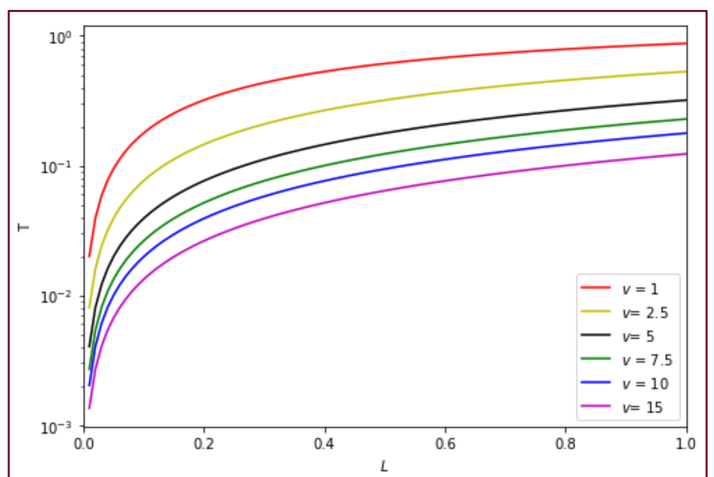

Figure 5: Conditional mean first passage time at constant death rate

The obtained expression for the conditional mean first passage time of evanescent run-and-tumble particles when the boundaries are symmetric, $a = -b = L$ is given by

$$T = \frac{2b/v + \epsilon b^2(2\lambda+\alpha)/v^2}{2\epsilon + 2b\lambda/v + \epsilon b^2\lambda(\alpha+\lambda)/v^2}.$$

Figure 4 shows the conditional mean first passage time of evanescent run-and-tumble particles with varying death rate at constant speed while Figure 5 is when the death rate is constant and the speed varies. Both plots show a decrease in the conditional mean first passage time when death rate (Figure 4) and speed (Figure 5) increases . The decrease in the conditional mean first passage time is consistent with the intuition that evanescent run-and-tumble particles must reach the absorbing boundaries at a lesser time so that they get absorbed before death takes place.

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